

Paper Reference 9FM0/3A
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 3A: Further Pure
Mathematics 1

Wednesday 19 June 2024 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE

Mathematical Formulae and Statistical
Tables (Green), calculator

YOU WILL BE GIVEN

Diagram Booklet
Answer Booklet

V75686RA



Pearson

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

Turn over

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 10 questions in this Question Paper.

The total mark for this paper is 75.

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams in case you need them.

Turn over

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. (a) Refer to the table for Question 1 in the Diagram Booklet.

Given that

$$y = \ln(3 + x^2)$$

complete the table in the Diagram Booklet with the value of y corresponding to $x = 3$, giving your answer to 4 significant figures.

(1 mark)

(continued on the next page)

1. continued.

In part (b) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(continued on the next page)

Turn over

1. continued.

(b) Use Simpson's rule with all the values of y in the completed table to estimate, to 3 significant figures, the value of

$$\int_2^5 \ln(3 + x^2) dx$$

(3 marks)

(continued on the next page)

Turn over

1. continued.

**(c) Using your answer to part (b)
and making your method clear,
estimate the value of**

$$\int_2^5 \ln \sqrt{(3 + x^2)} \, dx$$

(1 mark)

(Total for Question 1 is 5 marks)

Turn over

- 2. Use algebra to determine the values of x for which**

$$|x^2 - 2x| \leq x$$

(Total for Question 2 is 4 marks)

3. Use L'Hospital's rule to show that

$$\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = 0$$

(Total for Question 3 is 6 marks)

4. Refer to the information for Question 4 in the Diagram Booklet. It shows the Taylor series expansion of $f(x)$ about $x = a$

The curve with equation $y = f(x)$ satisfies the differential equation

$$\cos x \frac{d^2 y}{dx^2} + y^2 \frac{dy}{dx} + \sin x = 0$$

(continued on the next page)

4. continued.

Given that $\left(\frac{\pi}{4}, 1\right)$ is a stationary point of the curve,

(a) determine the nature of this stationary point, giving a reason for your answer.

(2 marks)

(b) Show that

$\frac{d^3y}{dx^3} = \sqrt{2} - 2$ at this stationary point.

(4 marks)

(continued on the next page)

Turn over

4. continued.

(c) Hence determine a series solution for y , in ascending powers of

$\left(x - \frac{\pi}{4}\right)$ up to and including

the term in

$\left(x - \frac{\pi}{4}\right)^3$, giving each

coefficient in simplest form.

(2 marks)

(Total for Question 4 is 8 marks)

5. $y = e^{3x} \sin x$

(a) Use Leibnitz's theorem to show that

$$\frac{d^4 y}{dx^4} = 28e^{3x} \sin x + 96e^{3x} \cos x$$

(6 marks)

(continued on the next page)

5. continued.

(b) Hence express $\frac{d^4 y}{dx^4}$ in the form

$$\mathbf{Re^{3x} \sin(x + \alpha)}$$

where R and α are constants to be determined,

$$\mathbf{R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}}$$

(3 marks)

(Total for Question 5 is 9 marks)

Turn over

6. The ellipse **E** has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The hyperbola **H** has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where **a** and **b** are positive constants.

(continued on the next page)

6. continued.

Given that

- **the eccentricity of H is the reciprocal of the eccentricity of E**
- **the coordinates of the foci of H are the same as the coordinates of the foci of E**

determine

(i) the value of a

(ii) the value of b

(Total for Question 6 is 6 marks)

Turn over

- 7. In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(continued on the next page)

7. continued.

(a) Use the substitution

$t = \tan\left(\frac{\theta}{2}\right)$ to show that

$$\int \frac{1}{2\sin\theta + \cos\theta + 2} d\theta$$

$$= \int \frac{a}{(t+b)^2 + c} dt$$

**where a , b and c are constants
to be determined.**

(3 marks)

(continued on the next page)

Turn over

7. continued.

(b) Hence show that

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \frac{1}{2\sin\theta + \cos\theta + 2} d\theta = \ln\left(\frac{2\sqrt{3}}{3}\right)$$

(4 marks)

(Total for Question 7 is 7 marks)

8. The parabola **P** has equation $y^2 = 4ax$, where **a** is a positive constant.

The point

$A(at^2, 2at)$, where $t \neq 0$, lies on **P**

- (a) Use calculus to show that an equation of the tangent to **P** at **A** is

$$yt = x + at^2$$

(3 marks)

(continued on the next page)

Turn over

8. continued.

The point

$B(2k^2, 4k)$ and the point

$C(2k^2, -4k)$, where k is a constant,

lie on P

The tangent to P at B and the tangent to P at C intersect at the point D

Given that the area of the triangle BCD is 432

**(b) determine the coordinates of B
and the coordinates of C
(5 marks)**

(Total for Question 8 is 8 marks)

Turn over

9. (i) The line L_1 has equation

$$\underline{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$$

The line L_2 has equation

$$\underline{r} = \begin{pmatrix} 13 \\ 5 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$$

where λ and μ are scalar parameters.

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Turn over

9. (i) continued.

The lines L_1 and L_2 intersect at the point P

(a) Determine the coordinates of P

(2 marks)

Given that the plane Π contains both L_1 and L_2

(b) determine a Cartesian equation for Π

(4 marks)

(continued on the next page)

Turn over

9. continued.

(ii) Determine a Cartesian equation for each of the two lines that

- pass through $(0, 0, 0)$**
- make an angle of 60° with the x -axis**
- make an angle of 45° with the y -axis**

(4 marks)

(Total for Question 9 is 10 marks)

10. The motion of a particle **P** along the **X**–axis is modelled by the differential equation

$$t^2 \frac{d^2x}{dt^2} - 2t(t + 1) \frac{dx}{dt} + 2(t + 1)x = 8t^3 e^t \quad (\text{I})$$

where **P** has displacement **x** metres from the origin **O** at time **t** minutes,
 $t > 0$

(continued on the next page)

10. continued.

(a) Show that the transformation $x = tu$ transforms the differential equation (I) into the differential equation

$$\frac{d^2u}{dt^2} - 2 \frac{du}{dt} = 8e^t$$

(4 marks)

(continued on the next page)

Turn over

10. continued.

Given that P is at O when

$t = \ln 3$ and when

$t = \ln 5$

**(b) determine the particular solution
of the differential equation (I)
(8 marks)**

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
